BJS Federation of Schools

Calculation Policy 2021

Policy Adopted: Spring 2022

Signed: _____

Ms A. Parker Executive Headteacher

Contents

Introduction

The Calculation Policy has been put together in consultation with the staff at the BJS Partnership. All staff should refer to the policy in order to provide continuity and development in written calculations as children progress through the schools.

This policy brings together our ideas and practice in striving to provide our children with every opportunity to achieve in mathematics.

Aims:

- To raise attainment and increase pupil progress
- To have a consistent approach to the teaching of calculations throughout the school
- To ensure progression between classes and across the Key Stages
- To provide all teaching and support staff with a framework for the teaching of calculation strategies in line with the Maths Mastery approach.

How to use this policy:

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems

This policy has been designed to teach children through the use of concrete, pictorial and abstract representations (CPA).

Concrete representation: a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.

Pictorial representation: a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation: a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24.

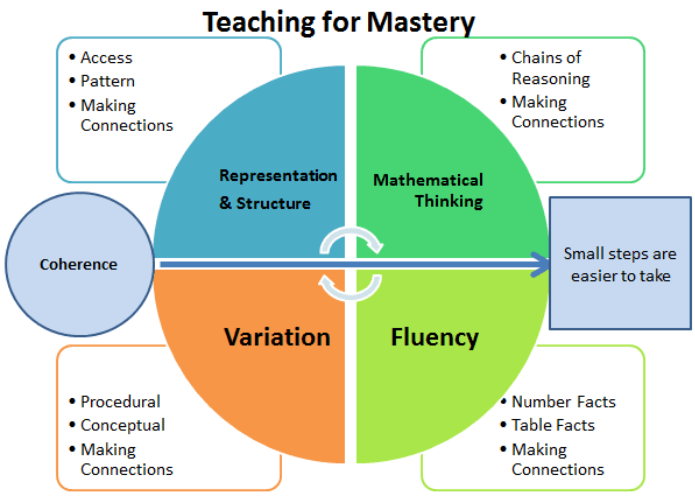
It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations. It is

also worth noting that if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or, if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial. Similarly, although the strategies are taught in a progressive sequence, they are designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts. So, as a new strategy is taught, it does not necessarily supersede the previous, but builds on prior learning to enable children to have a variety of tools to select from. As children become increasingly independent, they will be able to, and must be encouraged to, select those strategies which are most efficient for the task.

Mastery in Mathematics

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.

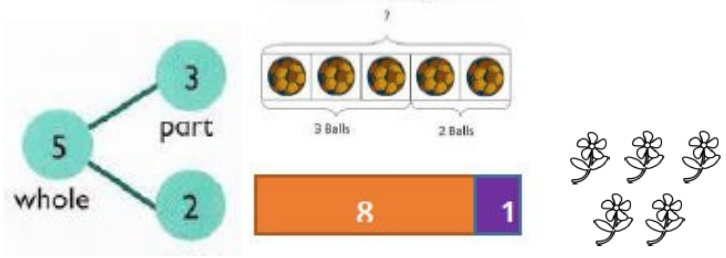
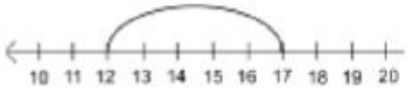
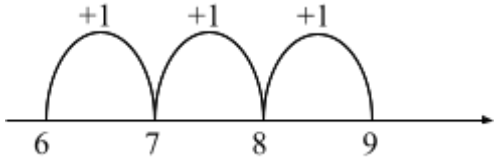
Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other.

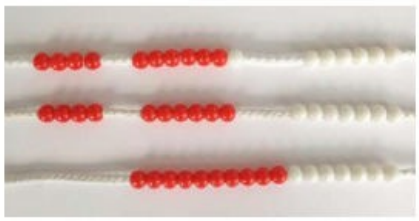
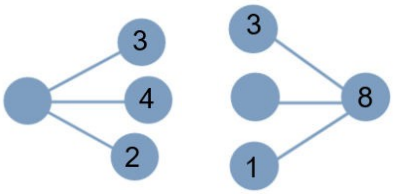
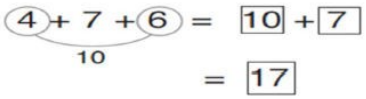
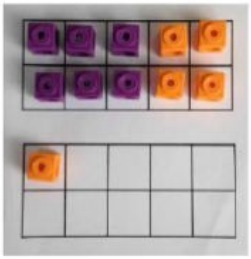
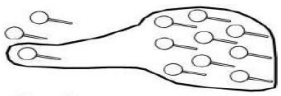


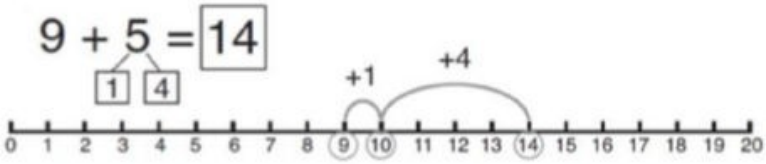
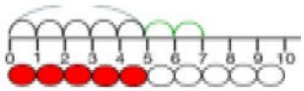
Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). In certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant, real objects, apparatus, pictures of diagrams) and explained carefully. High expectations of the mathematical language used are essential to close the vocabulary gap and ensure the children have mathematical cultural capital.

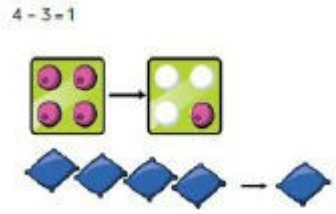
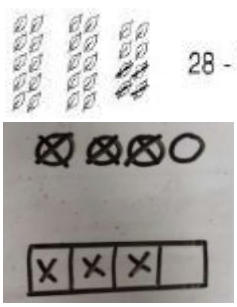
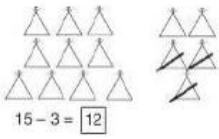
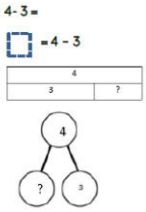
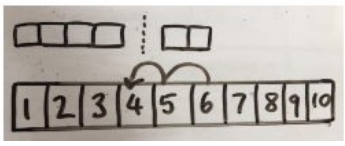
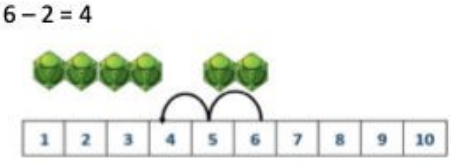
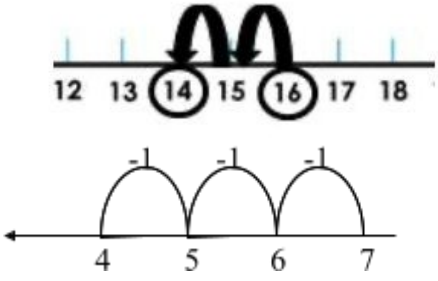
YEAR 1 - ADDITION

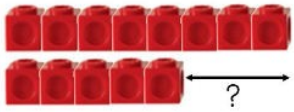
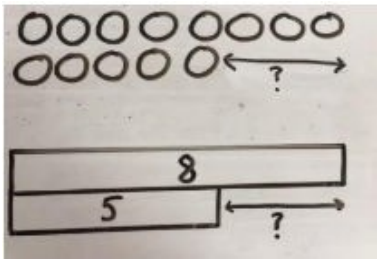
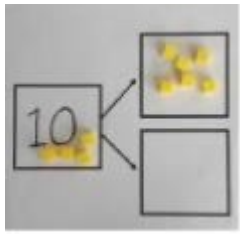
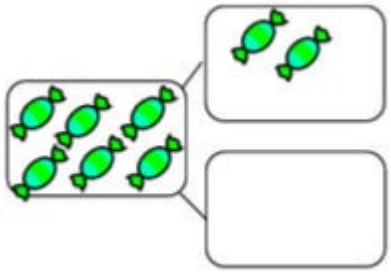
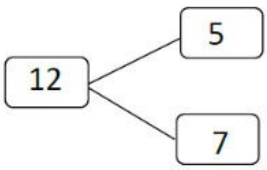
Objective:	CPA:
Combining two parts to make a whole number.	**Concrete**: Use cubes to add two numbers together as a group or a bar.  **Pictorial:** Use pictures to add two numbers together as a group or in a bar.  **Abstract:** Use the part-part whole diagram as shown above to move into the abstract. 4 + 3 = 7 10 = 6 + 4 Empty box with numerals 4 + ☐ is 7
Starting at the bigger number and counting on.	**Concrete**: Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  Progress to using a number line or track to count on. Eg. 4+ 2 = 6  **Pictorial**: Start at the larger number on the number line and count on in ones or in one jump to find the answer.  Eg. 12 + 5 = 17 Once children are confident with using a marked number line, introduce an empty number line to count on.  Eg. 6 + 3 = **Abstract:** Place the larger number in your head

	and count on the smaller number to find your answer. Mental strategies to solve simple problems in addition and encourage children's own recording using conventional signs.
Adding three single digit numbers (make ten first)	**Concrete:** Pupils may need to try different combinations before they find the two numbers that make 10. Eg. 4 + 7 + 6 =  The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten. The second bead string shows 4, 6 and then 7. The final bead string shows how they have now been put together to find the total. **Pictorial**: Progress to using part-part-part whole model.  **Abstract**: 3p + ☐ + 1p = 8p 3+4+2= ☐ 
Regrouping ten ones to make ten. *(this is an essential skill that will support column addition later on)*	**Concrete**: Start with the bigger number and use the smaller number to make 10.  Pupils should be encouraged to start at the greater number and partition the smaller number tomake ten. The colours of the beads on the bead string make it clear how many more need to be added to make ten.  Also, the empty spaces on the ten frame make it clear how many more are needed to make ten. **Pictorial**: Use pictures or a number line. Regroup or partition the smaller number to make 10.  

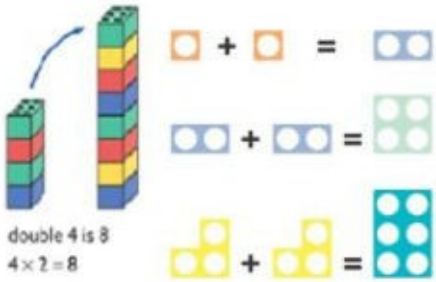
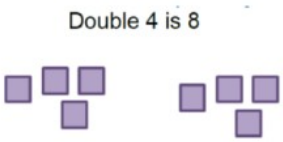
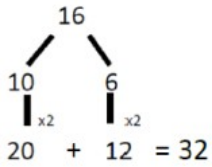
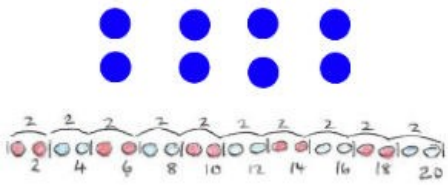
	 **Abstract:** 7 + 4 = 11 *"If I am at seven, how many more do I need to make 10? How many more do I add on now?"* Children are encouraged to use full sentences and support children using language structures on slides.
Represent & use number bonds and related subtraction facts within 20.	**Concrete:** E. 2 more than 5  **Pictorial:**   **Abstract:** Emphasis should be on the language: *"1 more than 5 is equal to 6"* *"2 more than 5 is 7"* *"8 is 3 more than 5"* Over time, pupils should be encouraged to rely more on their number bonds knowledge than counting strategies.

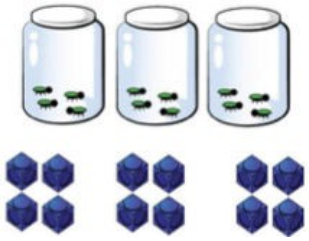
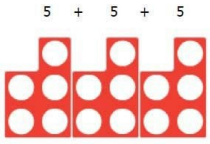
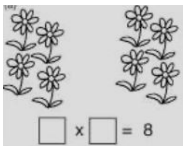
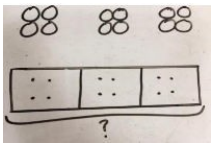
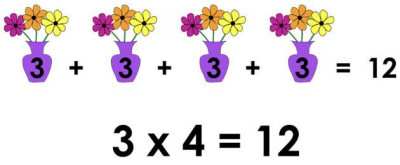
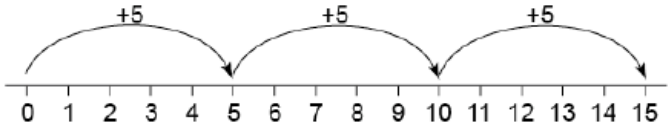
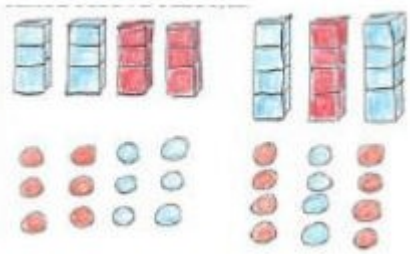
YEAR 1 - SUBTRACTION

Objective:	CPA:
Taking away from the ones.	**Concrete**: Use physical objects, counters, cubes etc, to show how objects can be taken away.  **Pictorial:** Cross out drawn objects to show how many have been taken away. The bar model can also be used.    **Abstract:** Part-part whole, missing numbers and the bar model can be used to support the pupils.
Counting back.	Pupils should be encouraged to rely on number bond knowledge as time goes on, rather than using counting back as their main strategy.  **Concrete:** Counting back (using number lines or number tracks) children start with 6 and count back 2. **Pictorial:** Children represent what they see pictorially.    **Abstract**: Children represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.

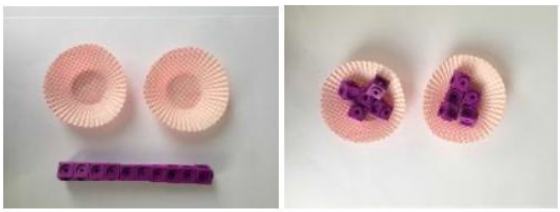
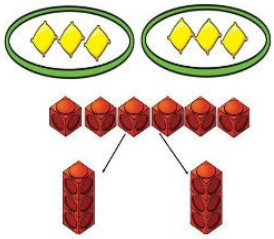
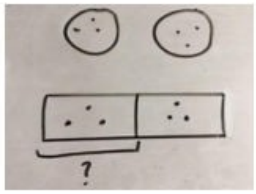
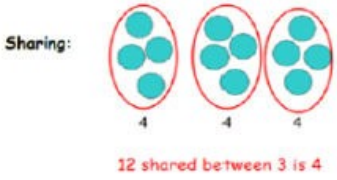
Finding the difference	**Concrete:** Finding the difference using cubes, Numicon, or other objects. Calculate the difference between 8 and 5: **Pictorial:** Children draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.   **Abstract:** Find the difference between 8 and 5. *8 - 5, the difference is…* Children to explore why *9 - 6 = 8 - 5 = 7 - 4 have the same difference*
Represent and use number bonds and related subtraction facts within 20.	**Concrete:** Link to addition- use the part-part whole model to model the inverse. **Pictorial:** Use pictorial representation to show the parts.   **Abstract**: Move to using numbers within the part-whole model.  

YEAR 1 - MULTIPLICATION

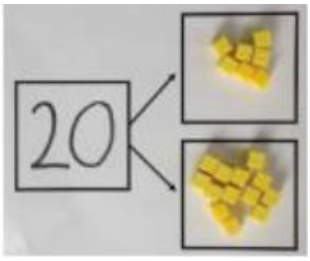
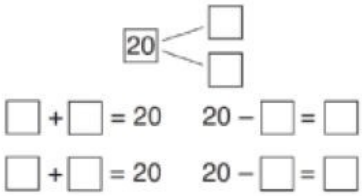
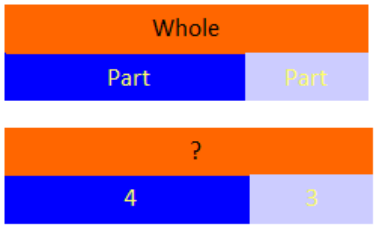
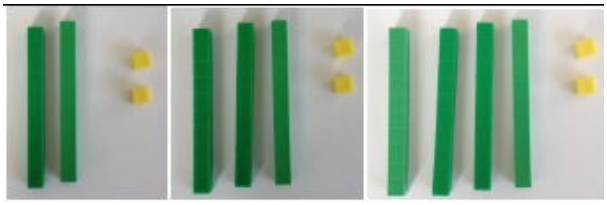
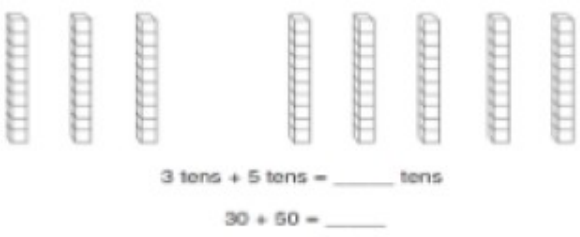
Objective:	CPA:
Doubling numbers.	**Concrete:** Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.   **Pictorial:** Draw pictures to show how to double numbers. **Abstract**: Partition a number and then double each part before recombining it back together. 
Counting in multiples. *(skip counting)*	The representation for the amount of groups supports pupils' understanding of the written equation. Eg two groups of 2 are 2,4. Or five groups of 2 are 2,4,6,8,10. Number lines can be used in the same way as the bead string   **Concrete:** Count the group as children are kip counting, children may use their fingers to help. **Pictorial:** Children make representations to show counting in multiples.  **Abstract:** Count in multiples of a number aloud. Write sequences with multiples of numbers. *2, 4, 6, 8, 10* *5, 10, 15, 20, 25, 30*

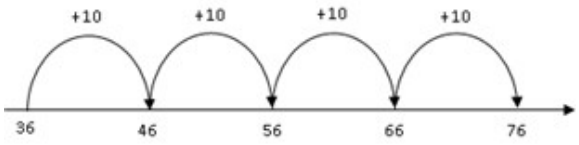
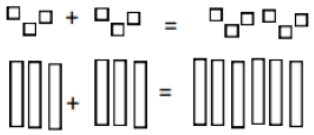
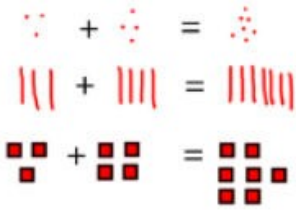
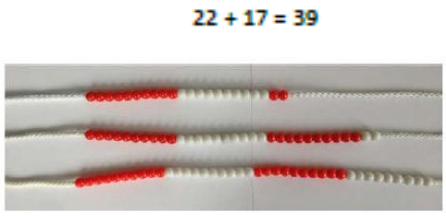
Repeated addition/repeated grouping	The importance should be placed on the vocabulary used alongside the calculation. **Concrete:** 3 x 4 = 4 + 4 + 4 = There are 3 equal groups, with 4 in each group.   **Pictorial:** Children to represent the practical resources in a picture and use a bar model.    **3 x 4 = 12** **Abstract:** 3 x 4 = 12 4 + 4 + 4 = 12 
Understanding arrays.	**Concrete:** Use objects laid out in arrays to find the answer to 2 lots of 5, 3 lots of 2s.   **Pictorial:** Draw representations of arrays to demonstrate understanding. **Abstract**: 3 x 2 = 6 2 x 5 = 10

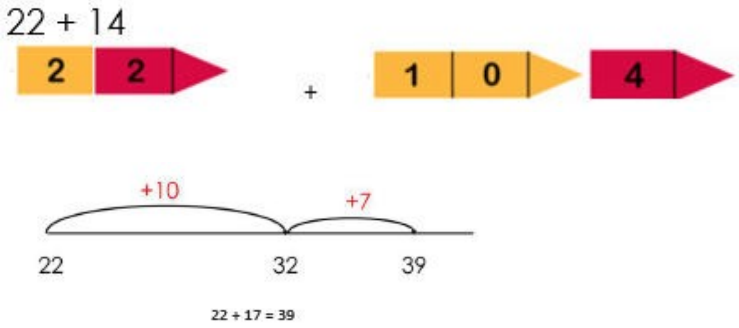
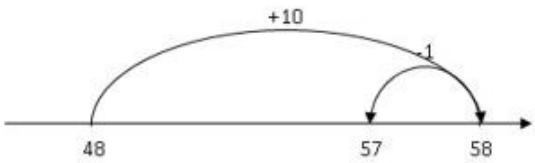
YEAR 1 - DIVISION

Objective:	CPA:
Division as sharing	**Concrete:** Sharing using a range of objects: 6 ÷ 2 =   **Pictorial:** Use pictures or shapes to share quantities:  **Abstract:** Children continue with the pictorial method until fully secure. Children should also be encouraged to use their 2 times tables facts. To progress further, children can then be moved onto: '6 shared between 2 is 3' 

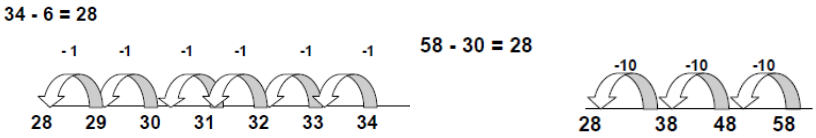
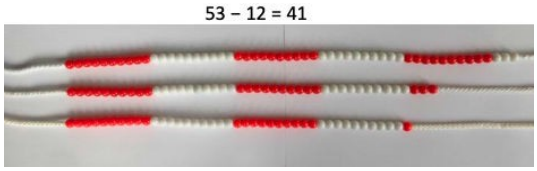
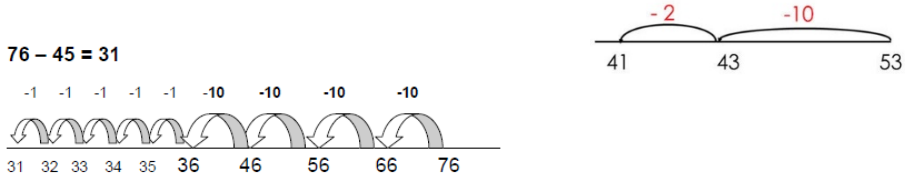
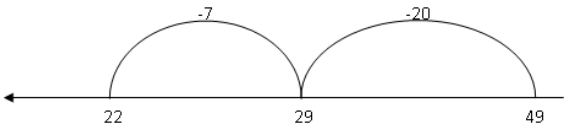
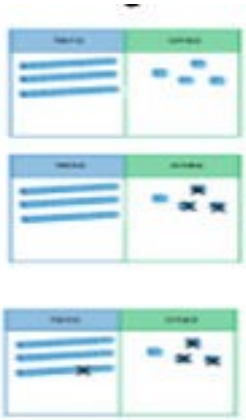
YEAR 2 - ADDITION

Objective:	CPA:
Use known number facts including different combinations of tens & ones of any 2 digit number. Part-part-whole.	Pupils explore the different ways of making 20. They can do this with all numbers using the same representations. **Concrete**: Children to explore ways to make numbers.  **Pictorial:** This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.   **Abstract**: Include teaching of the inverse of addition and subtraction: □ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Adding multiples of ten.	**Concrete**: Model using dienes and bead strings. 50 = 30 + 20   Exploring that one's digit does not change. **Pictorial:** Use representations for base ten. 

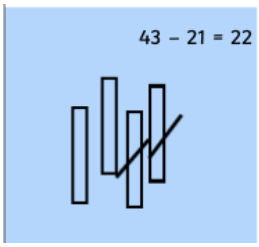
	 **Abstract**: 20 + 30 = 50 27 + 10 = 37 70 = 50 + 20 27 + 20 = 47 40 +___= 60 27 +___= 57
Use known facts to create derived facts.	**Concrete:** Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.  **Pictorial**: Children draw representations of H, T & O.  **Abstract:** 3+ 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 + 700
Partitioning one number, then adding tens and ones	**Concrete**: Model using dienes, place value counters and numicon.   Pupils can choose themselves which of the two numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown. Use arrow cards and place value materials to partition (and part partition) numbers into tens and units to add.

	Pictorial:  **Abstract:** Apply increasing knowledge of written method through recording addition of 1 and 2 digit numbers as vertical calculations. First using an expanded method with no exchanges. 22 + 14 = 2 0 + 2 + 1 0 + 4 3 0 + 6 =36 54 + 5 = 5 0 + 4 + 5 5 0 + 9 =59 Then shortening this to a more compact expanded method once children are secure with the place of value digit with no exchanges
Round and adjust.	Adding 9 by adding 10 and adjusting, then move to adding 19 by adding 20 then adjusting. **Concrete:** Children explain subtracting one using bead strings, numicon and dienes.  **Pictorial:**  **Abstract:** 48 + 9 = 35 + 9 = 48 +19 = 35 + 19 = Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.

YEAR 2 - SUBTRACTION

Objective:	CPA:
Subtracting tens and ones.	In year 1 pupils have been taught to partition the second number for this strategy as partitioning both numbers can lead to errors if regrouping is required. Children begin subtracting ones from a two digit number. Then move on to subtracting tens before subtracting tens and ones  **Concrete:** Pupils use bead strings, numicon, dienes or other objects to explore subtracting two digit numbers from two-digit numbers.  **Pictorial:** Then combining these (in conjunction with using a hundred square to show jumps in tens and ones)  This then becomes more efficient jumps along a number line.  **Abstract:** 76 - 45 =
Partitioning to subtract - without regrouping. (friendly numbers)	**Concrete:** Use dienes to show how to partition the number when subtracting without regrouping. 34 - 13 = 21 

Pictorial: Children draw representations of dienes and cross off.



Abstract: Applying increasing knowledge of written method through recording subtraction of 1 and 2 digit numbers as vertical calculations.

First using expanded method (with no regrouping))
78 - 3 =

	7	0	8
-			3
	7	0	5

=75

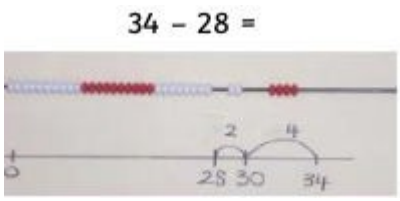
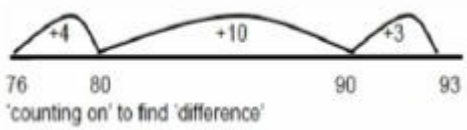
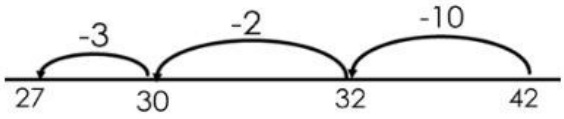
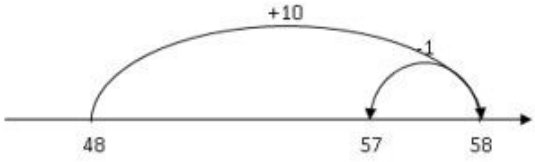
54 - 12 =

	5	0	4
-	1	0	2
	4	0	2

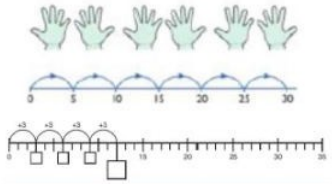
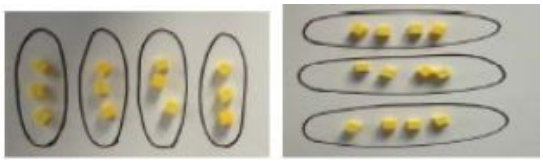
= 42

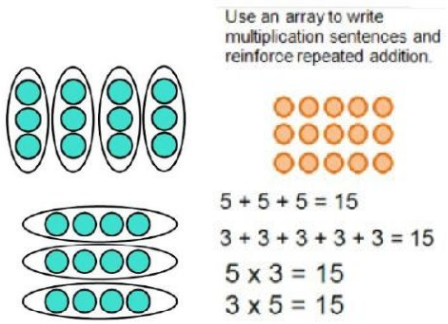
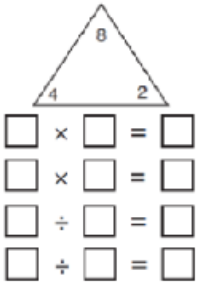
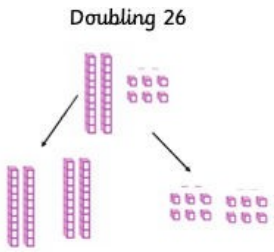
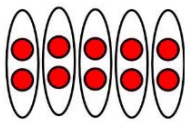
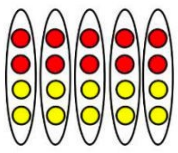
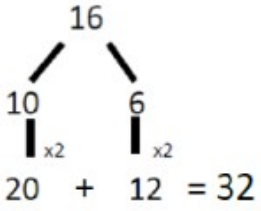
Moving to a compact method, once children are secure with the value of each digit (with no crossing the boundaries/regrouping)

	7 8	5 4
-	3	- 1 2
	7 5	4 2

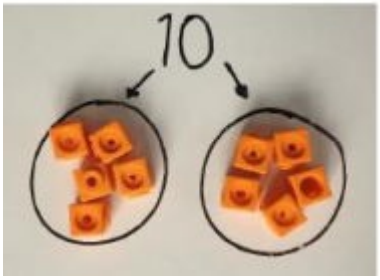
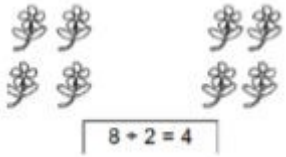
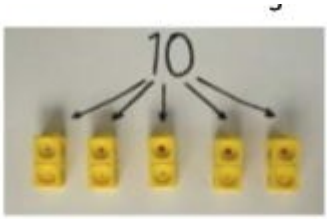
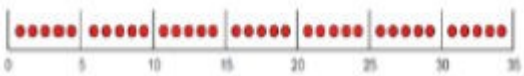
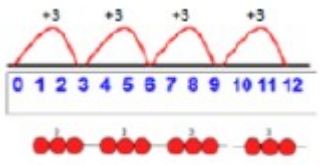
Making ten. *Crossing one ten, crossing more than one ten, crossing the hundreds.*	**Concrete:** Use a bead string or model counting to the next ten and the rest.   **Pictorial:** Use a number line to count on to the next ten and then the rest.   This strategy relies on an understanding that numbers can be partitioned in different ways in order to subtract to a multiple of ten. Pupils should develop an understanding that the parts can be added in any order. **Abstract:** 93 - 76 = 17
Round and adjust.	Adding 9 by adding 10 and adjusting, then move to adding 19 by adding 20 then adjusting. **Concrete:** Children explore adding 10 then subtracting one using bead strings, numicon and dienes.  **Pictorial:**  **Abstract:** 48 + 9 = 35 + 9 = 48 +19 = 35 + 19 = Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.

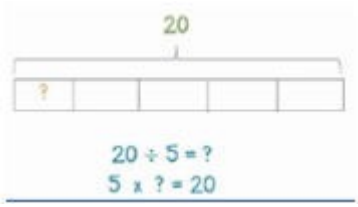
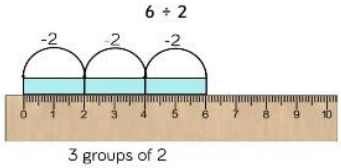
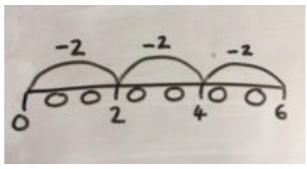
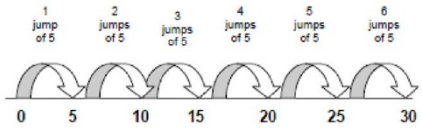
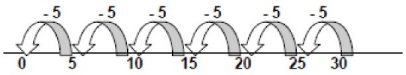
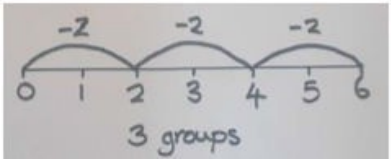
YEAR 2 - MULTIPLICATION

Objective:	CPA:
Counting in multiples of 2, 5 and 10 from 0 *(skip counting and repeated addition)*	Pupils can use their fingers as they count to develop an understanding of 'groups of'.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40 **Concrete:** Count the groups as children skip counting, children may use their fingers to help. Progress onto bar models.   **Pictorial:** Number lines, counting sticks and bar models should be used to show representations of counting in multiples.  **Abstract:** Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15, 0, 5, 10, 15, 20, 25, 30
Multiplication is commutative - arrays to represent multiplication equations	**Concrete**: Create arrays using counters, cubes and numicon.  Pupils should understand that an array can represent different equations and that as multiplication is commutative, the order of the multiplication does not change the answer.  **Pictorial:** Use representations of arrays to show different calculations and explore commutativity.

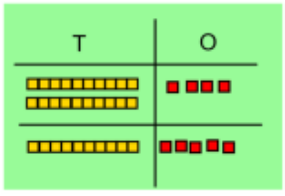
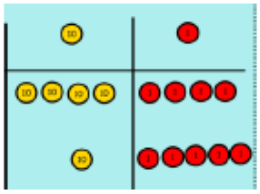
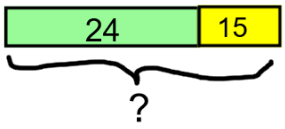
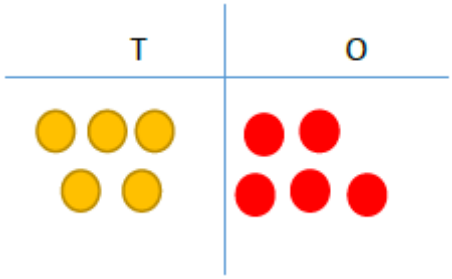
	 **Abstract:** 12 = 3 x 4 3 x 4 = 12 12 = 4 x 3 3 x 4 = 12
Using the inverse	This should be taught alongside division, so pupils learn how the two operations work alongside each other **Concrete:** Children use numicon, cubes or other objects to find out how division and multiplication work alongside each other.  **Pictorial**: Use the inverse triangle to create number sentences.  **Abstract:** Show all 8 related fact family sentences. 2 x 4 = 8 8 = 2 x 4 4 x 2 = 8 8 = 4 x 2 8 ÷ 2 = 8 2 = 8 ÷ 4 8 ÷ 4 = 2 4 = 8 ÷ 2
Doubling numbers.	At this stage they double the 2x table facts to derive the 4x table facts. **Concrete:** Model doubling using dienes and place value counters. **Pictorial:** Draw pictures and representations to demonstrate how to double numbers.    **Abstract:** Partition a number and then double each part before recombining it back together. 

YEAR 2 - DIVISION

Objective:	CPA:
Division as sharing	**Concrete:** I have 10 cubes, can you share them into 2 equal groups?  **Pictorial:** Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding: 12 ÷ 4 = 3  **Abstract:** 12 ÷ 3 = 4
Division as grouping	**Concrete**: Divide quantities into equal groups. Use cubes, counters, objects or place value counters to air understanding.   **Pictorial:** Use number lines for grouping:  Use bar model to support with division:

	 **Abstract:** 28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division using repeated subtraction	**Concrete:** Repeated subtraction using Cuisenaire rods above a ruler:  **Pictorial**: Children to represent repeated subtraction pictorially: **30 ÷ 5 = 6** 'How many jumps of five make thirty?'   **Also jump back to make the link with repeated subtraction:** **30 ÷ 5 = 6** 'How many groups of five?'  **Abstract:** Abstract number line to represent the equal groups that have been subtracted: 

YEAR 3 - ADDITION

Objective:	CPA:
Add 1 digit number to a 2 or 3 digit number.	Refer to **Year 3 Addition Calculation policy**
Column Addition- no regrouping (friendly numbers) (Up to 3 digit numbers)	**Concrete:** Add together the ones first then add the tens. Children to partition the number *'How many tens do we have? How many ones do we have?'* 63 + 32 = 95 6 tens is 60 and 3 ones 60 + 3 3 tens is 30 and 2 ones + 30 + 2 90 + 5 = 95 Use the Base 10 blocks first before moving onto place value counters. Pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulative or images. e.g. 'I know 2 + 1 = 3 so 2 tens plus 1 ten is equal to 3 tens which is 30.   **Pictorial:** After practically using the base 10 blocks and place value counters, children can draw the counters to help them solve additions.   **Abstract:** Add the ones first, then the tens, then the hundreds: 2 2 3 + 1 1 4 3 3 7 Children use written 'steps' to format their calculations.

Column Addition- with regrouping

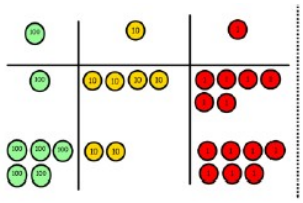
(Up to 3 digit numbers)

Concrete: Make both numbers on a place value grid.

146 + 527 =

1 hundreds 4 tens and 6 ones 100 + 40 + 6

5 hundreds 2 tens and 7 ones 500 + 20 + 7

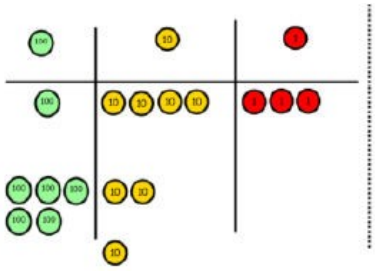


146
+ 527

Add the ones and exchange 10 ones for one ten.

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

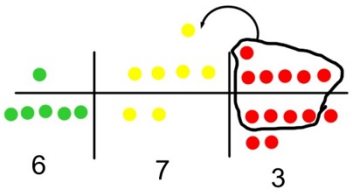
This can also be done with Base 10 to help children clearly see that 10 ones is equal to 1 ten and the 10 tens equal 100.



146
+ 527

As children move on to decimals, money and decimal place value, counters can be used to support learning.

Pictorial: Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Abstract: Children follow written steps to regroup and form calculations correctly.

		1	
	H	T	O
	1	4	6
+	5	2	7
	6	7	3

Always start in the ones coloumn and work to the left

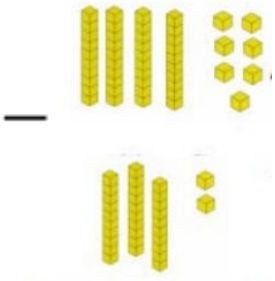
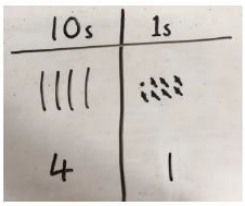
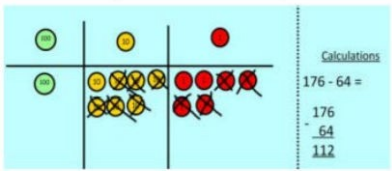
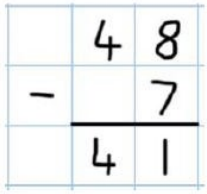
As the children move on, introduce decimals with the same number of decimal places and different. Money is used for context.

	72.8	
+	54.6	
	127.4	
	1 1	

	£	2	3	. 5	9
+	£		7	. 5	5
	£	3	1	. 1	4
		1	1	1	

	2	3	.	3	6	1
		9	.	0	8	0
	5	9	.	7	7	0
+		1	.	3	0	0
	9	3	.	5	1	1
	2	1		2		

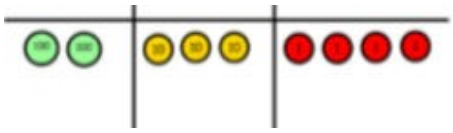
YEAR 3 - SUBTRACTION

Objective:	CPA:
Column subtraction - without exchanging. (friendly numbers)	**Concrete:** Column method using base ten or numicon.  Subtract the ones first then subtract the tens. Children to partition the number 'How many tens do we have? How many ones do we have?' 47 + 32 = 4 tens is 40 and 7 ones 3 tens is 30 and 2 ones 40 + 7 - 30 + 2 10 + 5 = 15 Use the Base 10 blocks first before moving onto place value counters and numicon. Pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulative or images. e.g. *'I know 4 - 3 = 1 so 4 tens take away 3 tens is equal to 1 tens which is 10.'*  **Pictorial:** Children to represent the base 10 pictorially- crossing out the ones, then the tens.  **Abstract:** Children should begin with the expanded form before moving on to the formal written method. Children use written steps to format the question correctly. 268 - 124 = 200 + 60 + 8 100 + 20 + 4 100 + 40 + 4 
Column subtraction - with exchanging	**Concrete:** Begin with base 10 or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.  Column method (using base 10 and having to exchange) 45 - 26

1. Start by partitioning 45
2. Exchange one ten for ten more ones
3. Subtract the ones, then the tens

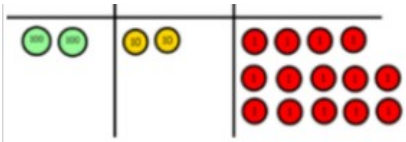
Use base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters.



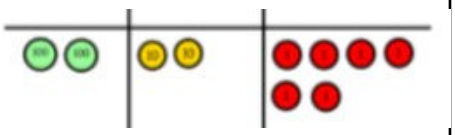
2 3 4
- 8 8

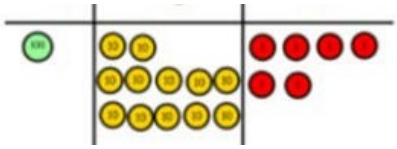
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Now I can subtract my ones.

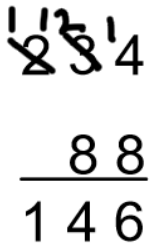
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



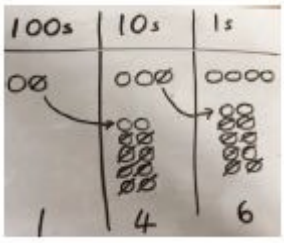


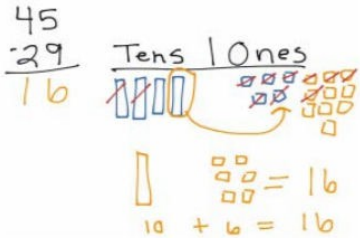
Now I can take away eight tens and complete my subtraction.

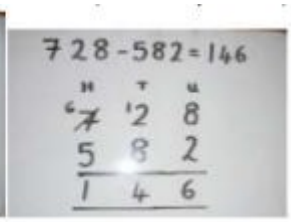
Show children how the concrete method links to the written method alongside your working. cross out the numbers when exchanging and show where we write out a new amount.



Pictorial: Represent the place value counters pictorially; remembering to show what has been exchanged.

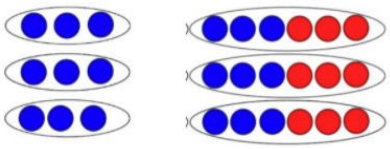
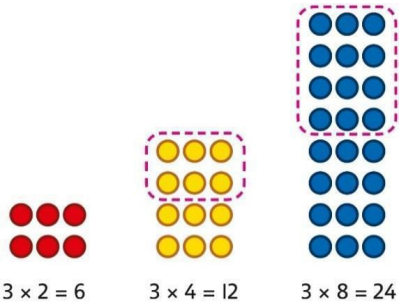
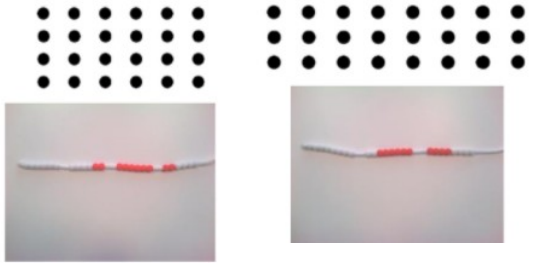


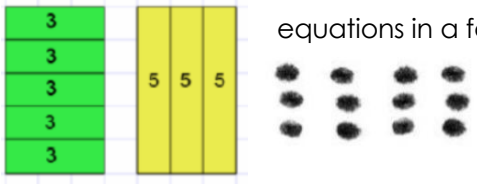
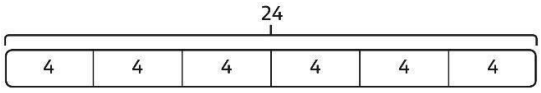
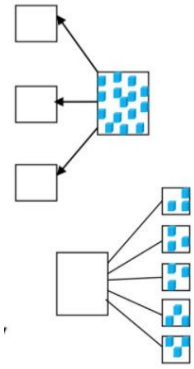
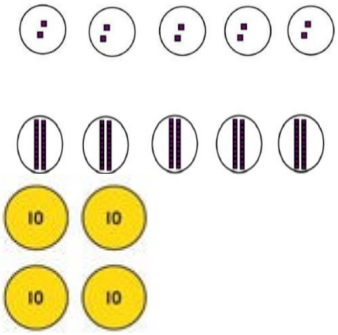
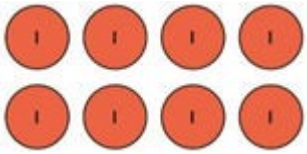
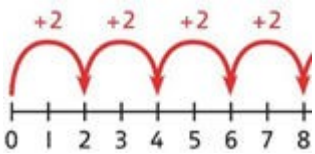




Abstract: Children should begin with the expanded form. Moving onto the written method.

YEAR 3 - MULTIPLICATION

Objective:	CPA:
Doubling to derive new multiplication facts	**Concrete:** Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy. This builds on the doubling strategy from year 2. **Pictorial:** Draw pictures and arrays to support understanding 3 x 9 = 3 x 6 = double 9 =   **Abstract:** If 3 x 8 = 24 what is 6 x 8? How do you know this?
Counting in multiples of 2, 3, 4, 5. 6, 8 and 10 *(skip counting and repeated addition)*	Refer to **Year 2 multiplication calculation policy**.  Rehearsal of previously learnt tables from Year 2 as well as new content for Year 3 should be incorporated into transition activities and practised regularly. 6 lots of 4 4, 8, 12, 16, 20, 24 4 + 4 + 4 + 4 + 4 +4= 8 lots of 3 8 groups of 3 3, 6, 9, 12, 15, 18, 21, 24 3+ 3+ 3+ 3+ 3+ 3+ 3+3=
Multiplication is commutative *(part part- whole model, arrays and bar model to establish commutativity and inverse relationship between multiplication and division)*	Refer to **Year 2 multiplication calculation policy.** **Concrete:** Pupils continue to use arrays with counters, cubes and numicon. Use blank Part-Part-Whole and move manipulatives. Use language structures and encourage children to use. E.g 1 group of 3, 2 groups of 3, 3 groups of 3, 4 groups of 3 make 12 12 has been shared equally into 4 groups I have 3 in each group. 4 groups of 3 make 12.

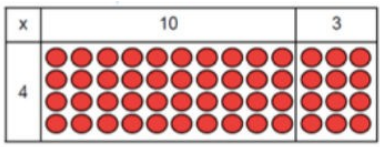
	Pictorial: Use representations of arrays, part part whole and bars models to identify all the equations in a fact family.    **Abstract:** 3 x 4 = 12 12 ÷ 3 = 4 3 lots of 4 = 12 4 x 3 = 12 12 ÷ 4 = 3 4 lots of 3 = 12 There are___altogether 12 divided by 3 = 4 There are____groups. 12 divided by 4 = 3 There are____in each group 3 x___= 12 ___÷ 3 = 4 _ x 4 = 12 12 ÷___= 4
Ten times bigger	**Concrete:** Pupil's work on this must be firmly based on concrete representations. The language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.   4 groups of 2 ones is 8 ones. 4 × 2 = 8 4 groups of 2 tens is 8 tens. 4 × 20 = 80 Understand how to use known times-tables to multiply multiples of 10. **Pictorial:**  4 × 2 = 8 4 × 20 = 80

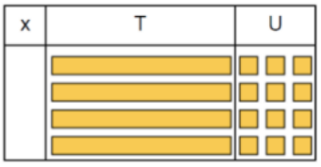
Multiplying 2 digit number by 1 digit number

(Grid method- partitioning)

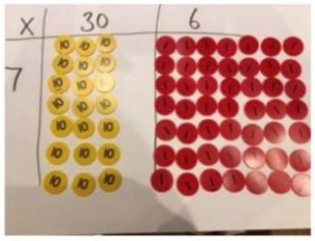
Concrete

Show the links with arrays to first introduce the grid method.





Move on to using Base 10 to move towards a more compact method.

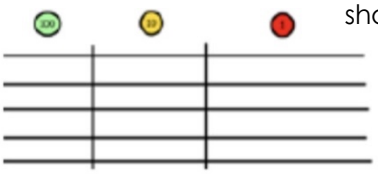


Move onto place value counters to show how we are finding groups of a number. Add up each column, starting with the ones making any exchanges needed.

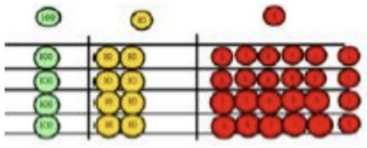
The calculation will be shown alongside the model chosen to see the connection.

e.g. 36 x 7

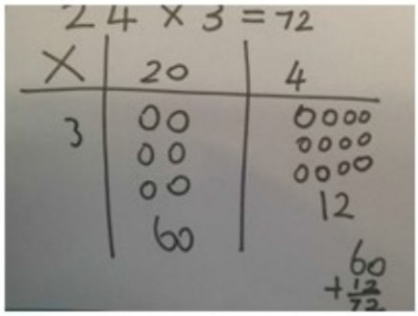
Ensure you make it clear if you are multiplying by 7 you will need 7 rows. If you are multiplying by 4, show 4 rows

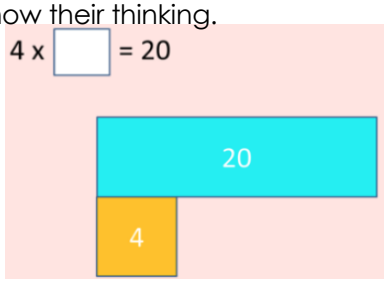


4 x 126



Pictorial: Children can represent their work with place value counters in a way that they understand. They can draw the counters using colour to show different amounts or just use the circles in the different columns to show their thinking.



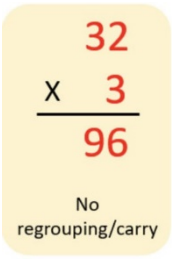


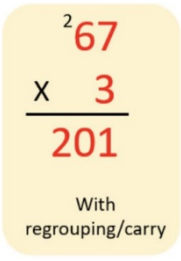
Bar models are used to explore missing numbers

X	10	3
8	80	24

Abstract: Begin with multiplying by one digit numbers and showing the clear addition alongside.

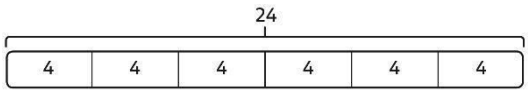
Introduce formal written method

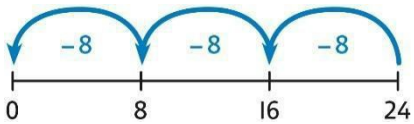


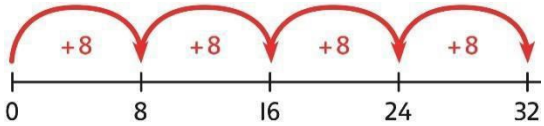


YEAR 3 - DIVISION

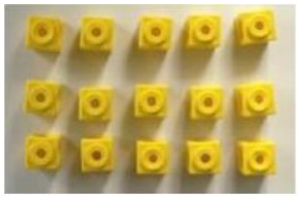
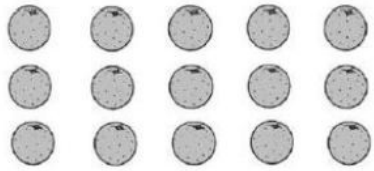
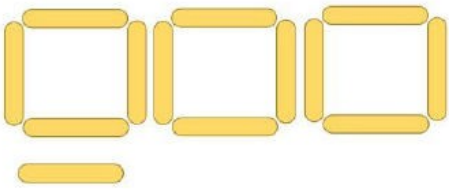
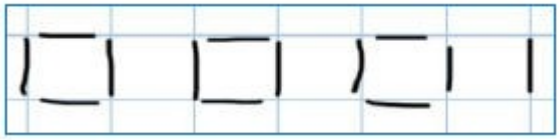
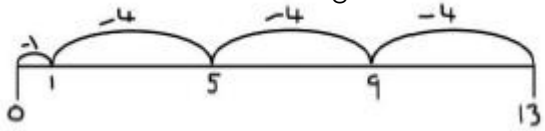
Objective:	CPA:
Using times-tables knowledge to divide (Make equal groups)	Use knowledge of known times-tables to calculate divisions. *'I need to work out 30 shared between 5.'* ***Concrete*:** Children use numicon, cubes or other manipulatives to show 5 groups of 3 or 3 groups of 5. Ask the children to share them equally from known multiplication facts *I know that 3 × 5 = 15* *so I know that 15 ÷ 3 = 5.* **Pictorial:** A bar model may represent the relationship between sharing and grouping. *24 ÷ 4 = 6* *24 ÷ 6 = 4* Children understand how division is related to both repeated subtraction and repeated addition. *24 ÷ 8 = 3* *32 ÷ 8 = 4* ***Abstract*:** Move on to sentence structures of knowing fact families. ___has been shared equally into____equal groups. I have_____in each group. _____groups of_____make _____ Introduce formal written method to support the number sentence. *24 divided by 4 equals 6.* *How many 4s are there in 24?*



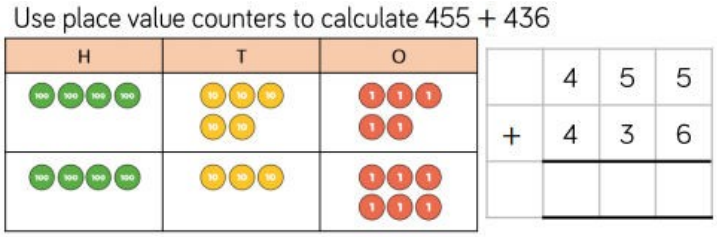
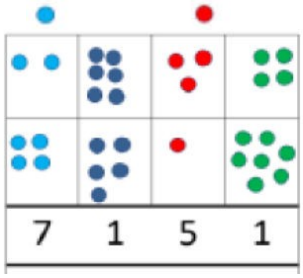
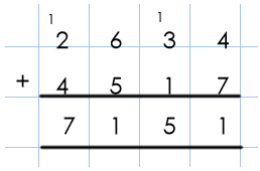
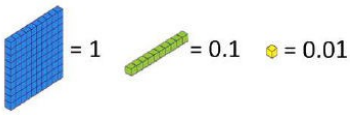
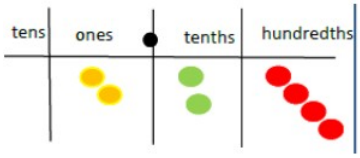




$$\begin{array}{r} 6 \\ 4\overline{)\,24} \end{array}$$

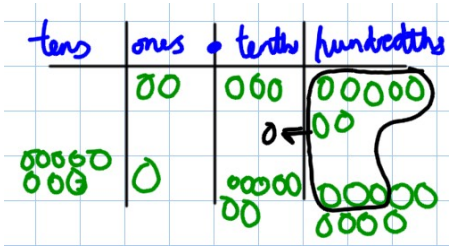
Division with arrays	**Concrete:** Link division to multiplication by creating an array and thinking about the number sentences that can be created.  *15 ÷ 3 = 5* *5 x 3 = 15* *15 ÷ 5 = 3* *3 x 5 = 15* **Pictorial:** Draw an array and use lines to split the array into groups to make multiplication and division sentences.  *15 ÷ 3 = 5* *5 x 3 = 15* *15 ÷ 5 = 3* *3 x 5 = 15* ***Abstract:*** *Find the inverse of multiplication and division sentences by creating eight linking number sentences:* *7 x 4 = 28* *4 x 7 = 28* *28 ÷ 7 = 4* *28 ÷ 4 = 7* *28 = 7 x 4* *28 = 4 x 7* *4 = 28 ÷ 7* *7 = 28 ÷ 4*
Division with remainders	**Concrete:** This can be done with lollipop sticks or Cuisenaire rods:  13 ÷ *4* Use of lollipop sticks to form whole-squares are made because we are dividing by 4. *There are 3 whole squares, with 1 left over.* **Pictorial**: Children represent the lollipop sticks pictorially. *There are 3 whole squares, with 1 left*  **Abstract:** 13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.  *3 groups of 4, with 1 left over*

YEAR 4 - ADDITION

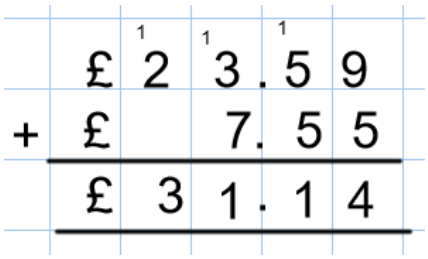
Objective:	CPA:
Formal written method- with and without regrouping (up to 4 digit numbers)	Refer to **Year 3 addition calculation policy**. Break down into necessary steps needed. e.g. *Add two 3 digit numbers - not crossing 10 or 100 Add two 4 digit numbers - no regrouping* *Add two 3 digit numbers- crossing 10 or 100* *Add two 4 digit numbers- one regroup* *Add two 4 digit numbers- more than one regrouping* **Concrete:** Continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. The calculation to be shown alongside the manipulatives used to see the connection.   **Pictorial**: Draw representations using place value grid. Number sentences and formal written methods are also shown.  **Abstract:** Formal written method. Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Calculating with decimals (Up to 2 dp) *All of the calculation strategies for integers (whole numbers) can be used to calculate with decimals.*	Assign different values to dienes equipment. These can then be used to build a conceptual understanding of the relationship between these. Place value counters are another useful manipulatives for representing decimal numbers.  **Concrete**: Introduce decimal place value counters and model exchange for addition. 

Pictorial: Children draw counters and place value charts.

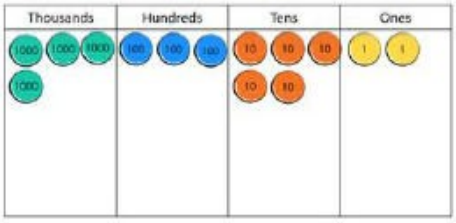
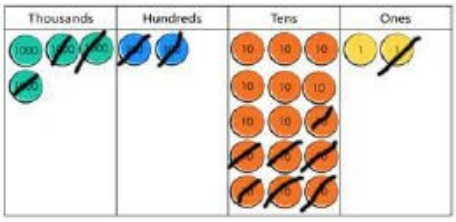
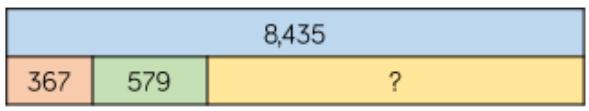
2.37 + 81.79



Abstract: As the children move on, introduce decimals with the same number of decimal places and different number of decimal places. Money and measures can be used here.

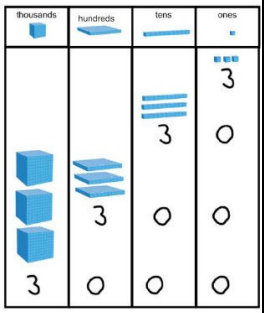
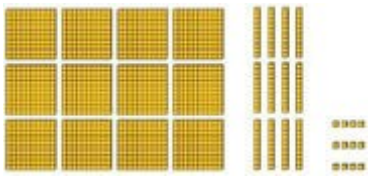


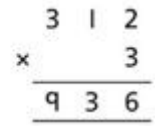
YEAR 4 - SUBTRACTION

Objective:	CPA:
Formal written method- with and without exchanging (Up to 4 digit numbers)	Refer to **Year 3 subtraction calculation policy.** Break down into necessary steps needed. e.g. *Subtract a 3 digit number from a 3 digit number- no exchange* *Subtract two 4 digit numbers- no exchange* *Subtract a 3 digit number from a 3 digit number- exchange Subtract two 4 digit numbers- one exchange* *Subtract two 4 digit numbers- more than one exchange* **Concrete:** Model process of exchange using numicon, base ten and then move to place value counters. Ensure the calculation is shown alongside the modern chosen.   $\begin{array}{r} 4\,\cancel{3}^{2}\,{}^{1}5\,2 \\ -\ 3\,2\,7\,1 \\ \hline 1\,0\,8\,1 \end{array}$ **Pictorial:** Children to draw place value counters and bar modelling and show their exchange- see Y3  Use bar modelling to support children with word problems **Abstract**: Formal written method. Children must understand what has happened when they have crossed out digits.- see Y3 What happens when you can't subtract 9 ones from 7 ones? What do we need to do? Why do we exchange? When do we exchange? $\begin{array}{r} \cancel{3}^{2}\,{}^{1}6\,{}^{7}\cancel{8}\,{}^{1}4 \\ -\ 2\,9\,1\,7 \\ \hline 7\,6\,7 \end{array}$

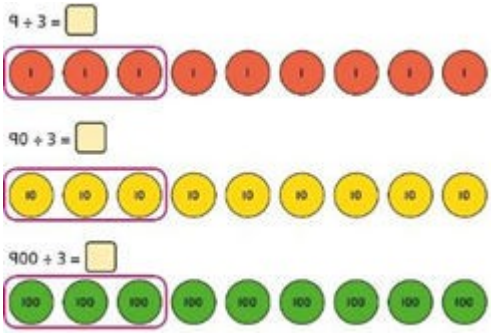
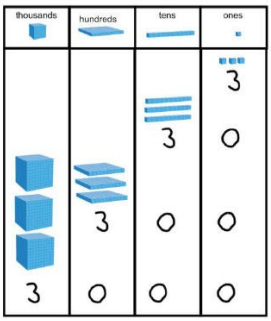
Calculating with decimals- with and without exchanging (Up to 2 dp) *All of the calculation strategies for integers (whole numbers) can be used to calculate with decimals*	Refer to **Year 3 subtraction calculation policy** and **Year 4 addition calculation policy** for CPA examples. In the context of money and measure.

YEAR 4 - MULTIPLICATION

Objective:	CPA:
Multiplying by multiples of 1, 10, 100.	**Concrete:** Use place value resources, dienes and counters to understand how to multiply by multiples of 1, 10 and 100. When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.  3 x 10 = 30 3 x 100 = 300 3 x 1000 = 3000 Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000.  3 lots of/ groups of 4 ones is 12 ones. 3 lots of/ groups of 4 tens is 12 tens. 3 lots of/ groups of 4 hundreds is 12 hundreds. *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200* ***Pictorial:*** Encourage children to draw the place value chart and show what is happening to each digit to support the use of concrete resources. What is the new value of the digit 3? **Abstract:** Use known facts and understanding of place value and commutativity to multiply mentally. Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact. Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts. 4 × 7 = 28 4 × 70 = 280 4 × 700 = 2,800 40 × 7 = 280 400 × 7 = 2,800
Times table knowledge up to 12 x 12	**Concrete:** Understand the special cases of multiplying by 1 and 0. Children use any resources to show the children the difference between x 1 and x 0.  *5 × 1 = 5* *5 × 0 = 0*

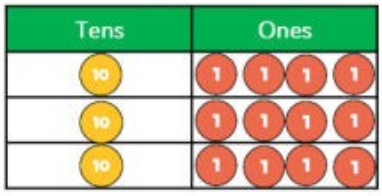
	Use numicon, counters or counters to represent the relationship between the times tables. E.g. x9 and x10   ×11 table and ×12 tables in relation to the ×10 table. *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8* **Pictorial:** To support concrete children to draw the cubes, using colours. **Abstract**: Move children on to writing full number sentences and knowing their times table mentally. Children being able to explain methods for learning their times tables, the relationships and difference between some multiplication facts.
Formal written method- Multiplying 2 digit and 3 digit numbers by 1 digit number	Refer to **Year 3 multiplication calculation policy** to consolidate grid method with partitioning. Use same methods to solve 3 digit numbers by 1 digit numbers before moving on to formal written method. **Concrete:** Use place value counters to make the 3 digit number then repeat this by the number you are multiplying by. Make 4 × 136 using equipment. I can work out how many 1s, 10s and 100s.  There are 4 × 6 ones… 24 ones There are 4 × 3 tens … 12 tens There are 4 × 1 hundreds … 4 hundreds **Pictorial:** Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit. Refer back to the grid method and allow children to discover the connection.   **Abstract**: Children use formal written method with and without regrouping.

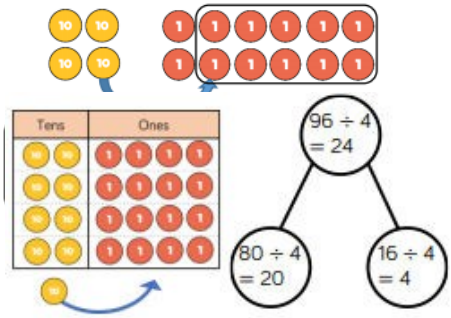
YEAR 4 - DIVISION

Objective:	CPA:
Dividing by multiples of 1, 10 and 100.	Refer to **Year 4 multiplication calculation policy** for CPA. When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller. When dividing multiples of ten, a placeholder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21 **Concrete:** Use dienes, place value charts and counters to show what is happening to each digit.   3 ÷ 1 = 3 30 ÷ 10 = 3 300 ÷ 100 = 3 300 ÷ 10 = 30 3000 ÷ 100 = 30 3000 ÷ 10 = 300 **Pictorial:** Encourage children to draw the place value chart and show what is happening to each digit to support the use of concrete resources. **Abstract:** Use known facts and understanding of place value and commutativity to multiply mentally. Emphasis is placed on understanding the relationship (10 times or 100 times smaller) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact. 9 ÷ 3 = 3 9 tens divided by 3 is 3 tens. 9 hundreds divided by 3 is 3 hundreds.
Formal written method- short division *(Up to 3-digit by 1-digit)*	Refer to **Year 1 and 2 division calculation policy** for CPA ideas. Refer to **Year 3 division calculation policy** to consolidate. Children divide 2-digit numbers by a 1-digit number by partitioning into tens and ones and sharing into equal groups. They divide numbers that involve exchanging between the tens and ones. Introduce remainders when consolidated. Then move on to 3-digit numbers by 1-digit numbers. rem

Concrete: Children use counters and place value charts to share the tens and ones.

42 ÷ 3 = 14 42 divided by 3





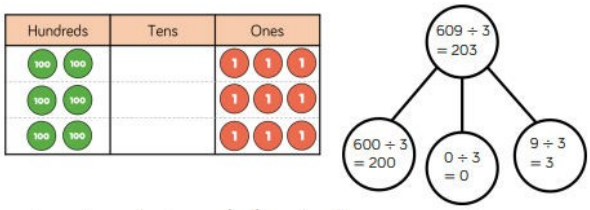
Pictorial: Children use known division facts from fact families to also use formal written method alongside using the counters and drawing them.

96 ÷ 4 =

E.g. 96 ÷ 4 = 24 80 ÷ 4 = 20

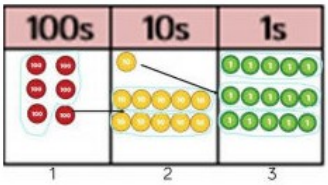
16 ÷ 4 = 4

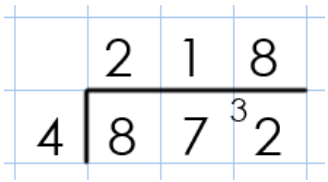
609 ÷ 3 =



615 divided by 5

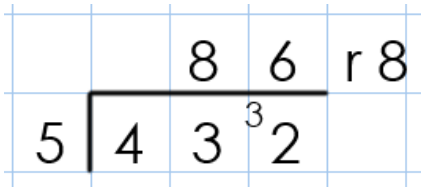
1. Make 615 with place value counters
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?



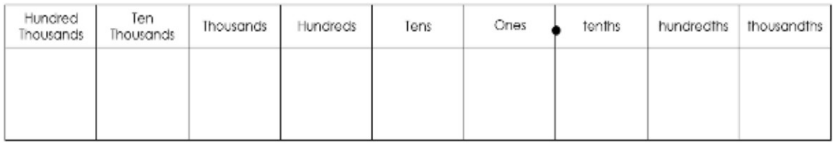
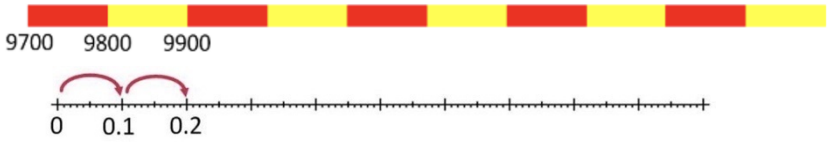
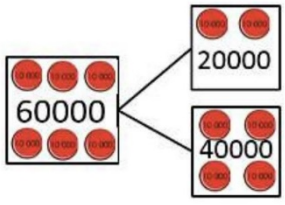
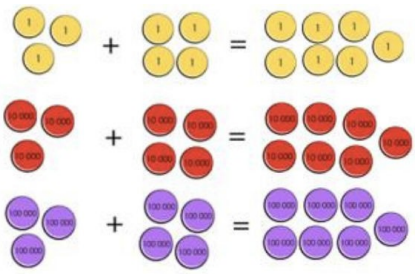


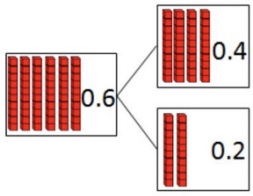
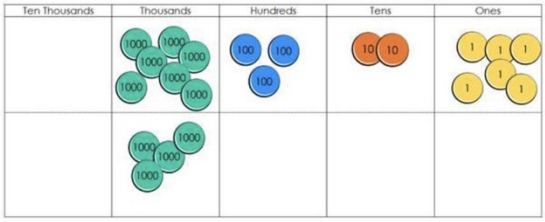
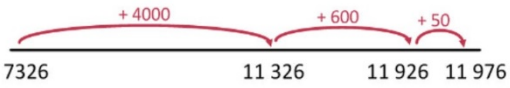
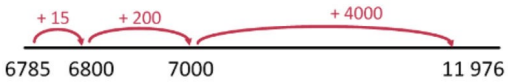
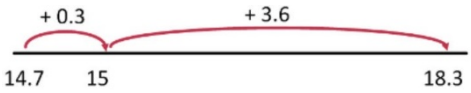
Abstract: Begin with divisions that divide equally with no remainders.

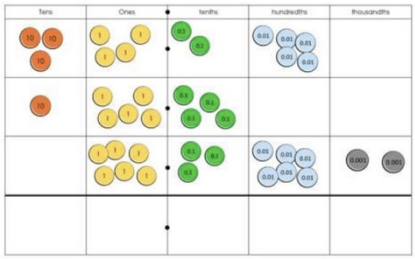
Move onto divisions with a remainder.



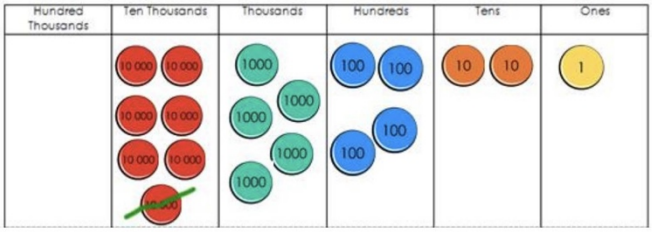
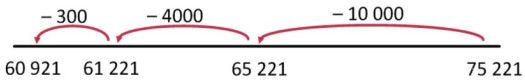
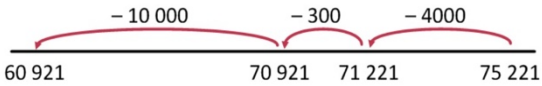
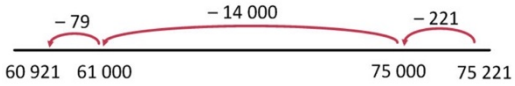
YEAR 5 - ADDITION

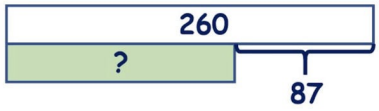
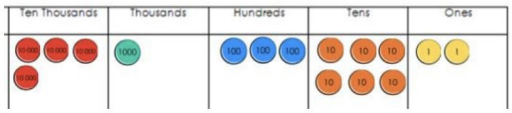
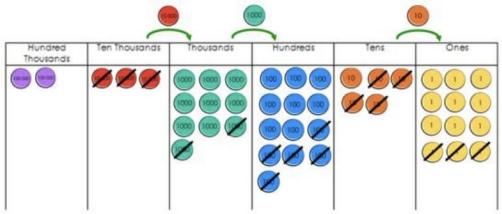
Objective:	CPA:
Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000 Adding and subtracting	Work with numbers up to 1,000,000 as well as tenths, hundredths and thousandths. **Concrete:** place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.  Counting sticks and number lines:  **Pictorial:** Children draw empty number lines and count forwards and backwards. **Abstract**: Children write number sentences. Pay particular attention to boundaries where regrouping happens more than once and so more than 1-digit changes. e.g. 9,900 + 100 = 10,000 or 99,000 + 1,000 = 100,000
Using known facts and understanding of place value to derive. *(multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths)* Adding and subtracting	Using the following language makes the logic explicit: *I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.* **Concrete:** 3 + 4 = 7 30 000 + 40 000 = 70 000 300 000 + 400 000 = 700 000  40 000 + 20 000 = 60 000 60 000 - 40 000 = 20 000 60 000 - 20 000 + 40 000  **Pictorial**: Encourage use of part-part-whole as well as drawing counters. 20 000 + 40 000 = 60 000

	 0.6 = 0.2 + 0.4 0.6 = 0.4 +0.4 0.2 = 0.6 - 0.4 0.4 = 0.6 - 0.2 **Abstract:** Number sentences.
Partitioning one number and applying known facts to add.	Partitioning into place value amounts (canonical partitioning) **Concrete:** Use place value counters to represent the larger number and then add each place value part of the other number.  4650 + 7326 = 7326 + 4000 + 600 + 50 **Pictorial:** Represent on an empty number line.  Extend the 'Make ten' strategy (See Y1 and Y2 addition) to count on to multiple of 10. Partitioning in different ways (non-canonical partitioning)  The strategy can be used with decimal numbers. 14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3  **Abstract:** Number sentences.
Formal written method- with and without regrouping. (Numbers with more than 4-digits and numbers with up to 3dp)	Refer to **Year 3 and Year 4 addition calculation policy**. Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective. Continue to use concrete manipulatives alongside the formal method. When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be

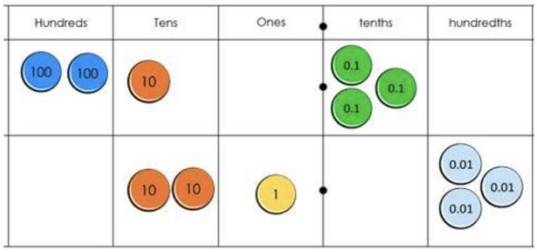
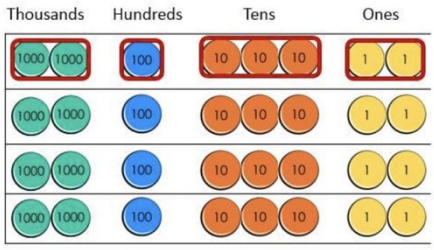
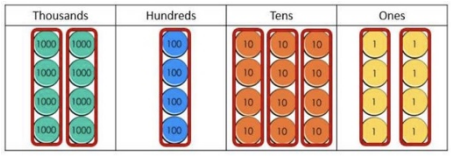
	encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.  34.25 15.4 + 6.362 56.012 34.250 15.400 + 06.362 56.012

YEAR 5 - SUBTRACTION

Objective:	CPA:
Using known facts and understanding of place value to derive. *(multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths)* Adding and subtracting	Refer to **Year 5 addition Calculation Policy.** 75 221 - 14 300 = 75 221 - 10 000 - 4000 - 300
Subtracting by partitioning and applying known facts.	Partitioning into place value amounts (canonical partitioning) **Concrete**: Use place value counters to represent the larger number.  **Pictorial:** Represent on an empty number line starting on the right and having the arrows jump to the left  Develop understanding that the parts can be subtracted in any order and the result will be the same.  Partitioning in different ways (non-canonical partitioning) Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.  Use Bar models represent subtractions in problem contexts, including 'find the difference'. Explain the mistake made when the columns have not been ordered correctly. Use approximation to check calculations. 'I calculated 18,000 + 4,000 mentally to check my subtraction.'

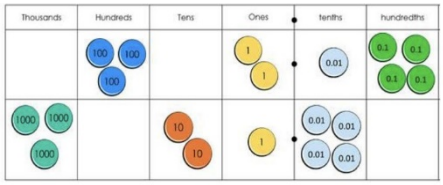
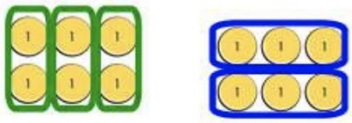
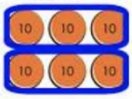
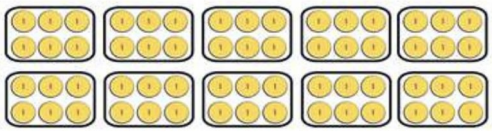
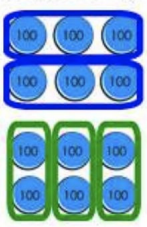
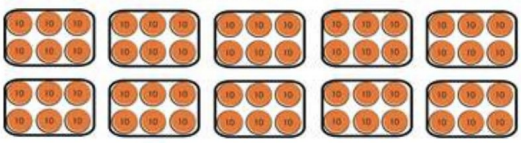
	 260 - 87 = **Abstract:** Number sentences
Formal written method- with and without exchanging (Numbers with more than 4-digits and numbers with up to 3dp)	Refer to **Year 3 and Year 4 subtraction calculation policy**. Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective. Continue to use concrete manipulatives alongside the formal method. 4 1 3 6 2 - 3 2 2 4 3  3 1 5 1 4 1 3 6 2 - 3 2 2 4 3 9 1 1 9 

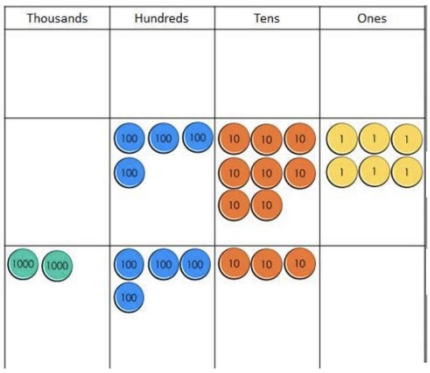
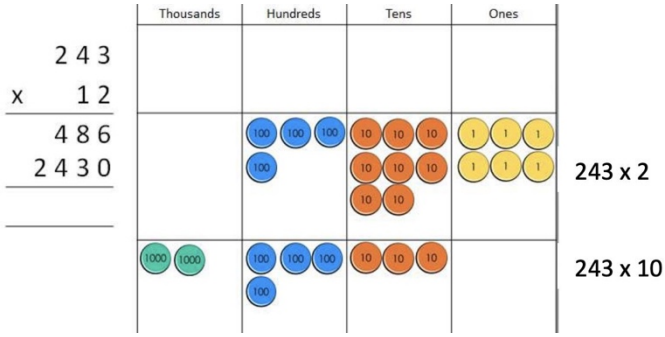
YEAR 5 - DIVISION

Objective:	CPA:
Dividing whole and those involving decimals by 10, 100 and 1,000	Refer to **Year 3 and 4 division calculation policy** and introduce dividing by 1,000 in the same CPA way. Introduce dividing numbers that involve decimals by 10, 100 and 100. When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller. When dividing multiples of ten, a placeholder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21 210.3 ÷ 10 = 21.03 
Formal written method- short division *(Up to 4-digit by 1-digit)* *The thought process of the traditional algorithm is as follows:* *How many 4s in 8? 2* *How many 4s in 5? 1 with 1 remaining so regroup.* *How many 4s in 12? 3* *How many 4s in 8? 2* *Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup. How many 4s in 8000? 2000 How many 4s in 500? 100 with 1 remaining (illogical)* *The answer would be 125.*	Refer to **Year 4 division calculation policy** and introduce dividing 4-digit by 1-digit in the same CPA way. Interpret remainders appropriately for the context. Children can then progress onto expressing the remainder as fractions (e.g. 5/8) and decimals (e.g. 663.625). **Concrete:**  <u>Sharing:</u> 8 thousands shared into 4 equal groups 5 hundreds shared into 4 equal groups Regroup 1 hundred for 10 tens 12 tens shared into 4 equal groups 8 ones shared into 4 equal groups. Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm. Using place value counters and finding groups of the divisor foreach power of ten will build conceptual understanding of the short division algorithm.  <u>Grouping:</u> How many groups of 4 thousands in 8 thousands? How many groups of 4 hundreds in 5 hundreds? 2 1 3 2 4 | 8 5 ^{1}2 8

	Regroup 1 hundred for 10 tens. How many groups of 4 tens in 12 tens? How many groups of 4 ones in 8 ones? **Pictorial**: Children draw the place value chart and counters and show if they are grouping or sharing. **Abstract:** Short multiplication with a clear understanding of regrouping. Using the language structures adobe to explain each step.

YEAR 5 - MULTIPLICATION

Objective:	CPA:
Multiplying whole and those involving decimals by 10, 100 and 1,000	Refer to **Year 3 and 4 multiplication calculation policy** and introduce multiplying by 1,000 in the same CPA way. Introduce multiplying numbers that involve decimals by 10, 100 and 100. When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater. 102.14 x 10 = 1021.4 
Using Known facts and place value to derive multiplication facts.	Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact. Knowledge of commutativity is further extended and applied to find a range of related facts Pupils should work with decimals with up to two decimal places. These derived facts should be used to estimate and check answers to calculations. **Concrete:** Use place value counters to show different arrays- 2 x 3 = 6 3 x 2 = 6  2 x 30 = 60  30 x 2 = 60   2 x 300 = 600 300 x 2 = 600 2 x 30 x 10 = 600 20 x 3 x 10 = 600  

	Pictorial: Children to draw place value counters to find larger fact families. **Abstract:** Number sentences. Multiplication facts pupils should be able to derive from known fact 2 100 000 | 700 000 x 3 | 70 000 x 30 | 7000 x 300 | 700 x 3000 | 70 x 30 000 | 7 x 300 000 210 000 | 70 000 x 3 | 7000 x 30 | 700 x 300 | 70 x 3000 | 7 x 30 000 21 000 | 7000 x 3 | 700 x 30 | 70 x 300 | 7 x 3000 2100 | 700 x 3 | 70 x 30 | 7 x 300 210 | 70 x 3 | 7 x 30 **21 = 7 x 3** 2.1 | 0.7 x 3 | 7 x 0.3 0.21 | 0.07 x 3 | 0.7 x 0.3 | 7 x 0.03 0.021 | 0.007 x 3 | 0.07 x 0.3 | 0.7 x 0.03 | 7 x 0.003
Formal written method (Multiplying 2-digit, 3-digit by 2 digit number)	Refer to **Year 3 and Year 4 multiplication calculation policy.** **Concrete:** Extend the place value chart model used in Year 4, using an additional row on the place value chart. **Pictorial:** Children draw representation alongside the calculation.   **Abstract:** Written form

YEAR 6 - ADDITION

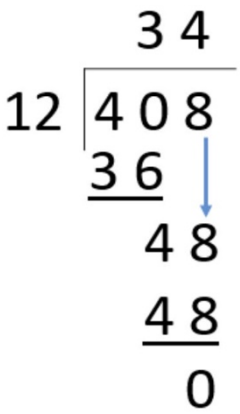
Objective:	CPA:
Count forwards or backwards in steps of powers of 10 for any given number up to 10,000,000	Refer to **Year 5 addition calculation policy** for CPA ideas for numbers up to 10 000 000.
Using known facts and understanding of place value to derive. *(multiples of 10 000, 100 000 and 1 000 000 as well as tenths, hundredths and thousandths))*	Refer to **Year 5 addition calculation policy** for CPA ideas for numbers up to 1 000 000
Partitioning one number and applying known facts to add.	Refer to **Year 5 addition calculation policy** for CPA.
Formal written method- with and without regrouping. (Numbers with more than 4-digits and numbers with up to 3dp)	Refer to **Year 3, 4 and 5 addition calculation policy** for CPA.

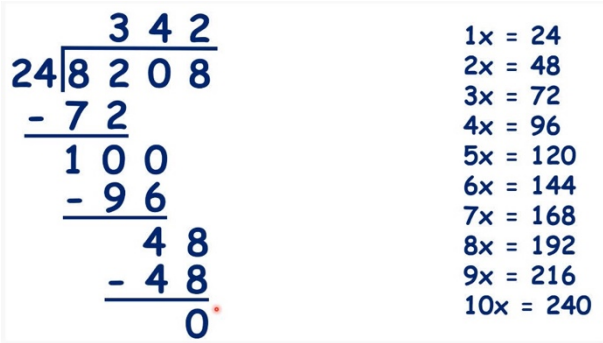
YEAR 6 - SUBTRACTION

Objective:	CPA:
Count forwards or backwards in steps of powers of 10 for any given number up to 10,000,000	Refer to **Year 5 addition calculation policy** for CPA ideas for numbers up to 10 000 000.
Using known facts and understanding of place value to derive. *(multiples of 10 000, 100 000 and 1 000 000 as well as tenths, hundredths and thousandths))*	Refer to **Year 5 addition calculation policy** for CPA ideas for numbers up to 1 000 000
Partitioning one number and applying known facts to add.	Refer to **Year 5 addition calculation policy** for CPA.
Formal written method- with and without regrouping. (Numbers with more than 4-digits and numbers with up to 3dp)	Refer to **Year 3, 4 and 5 addition calculation policy** for CPA.

YEAR 6 - DIVISION

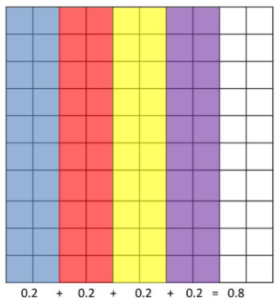
Objective:	CPA:
Dividing using formal written method- short division *(Up to 4-digit by 1-digit)*	Refer to **Year 4 and Year 5 division calculation policy**.
Dividing using formal written method- long division with and without remainders. *(4-digit by 2-digit)*	Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers. How many 12s go into 4, you can't do. Regroup the 4 which makes 40. How many 12s go into 40? You can make 36 with 3 lots of 12. Subtract 36 from 40 which leaves you with 4 tens. Bring the 8 ones down which makes 48. How many groups of 12 make 48? You can make 4 groups of 12 which is 48. 48 subtracted from 48 is 0 so we have no remainder. 408 ÷ 12 = 34. Move children on to larger numbers for the divisor. e.g. 8208 ÷ 24 How many 24s go into 8, you can't do. Regroup the 8 to make 82. How many 24s go into 82? Which multiple of 24 is closest to 82? You can make 72 which is 3 times with 10 remaining. Bring the 0 down to regroup 100. How many 24s go into 100? You can make 96 which is 4 lots of 24 with 4 remaining. Bring down the 8 to regroup 48. How many 24s make 48? 2 with no remainders.





YEAR 6 - MULTIPLICATION

Objective:	CPA:
Formal written method (Multiplying 2-digit, 3-digit and 4-digit numbers by 1 digit number)	Refer to **Year 4 and Year 5 multiplication calculation policy**.
Formal written method (Multiplying 2-digit, 3-digit by 2-digit number)	Refer to **Year 4 and Year 5 multiplication calculation policy**.
Multiply decimals by integers	Refer to **Year 3 and Year 4 multiplication calculation** policy for CPA. Apply similar CPA for Decimals. **Concrete:** use counters and place value charts. Multiply 1.212 by 3 **Pictorial:** Use 100 square to show children multiplying a decimal by integer is similar on repeated addition. Multiply 4 x 0.2. This means 4 groups of two tenths. 4 x 0.2 = 8 1.74 × 13 = 22.62



Policy compiled by:	Ms. A Parker	**Date:**	Spring 2022
Policy to be reviewed by: Spring 2024			
Signed Executive Headteacher:		**Date:**	_____